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INFLUENCE OF VIBRATIONAL RELAXATION ON THE PULSATION ACTIVITY IN FLOWS OF AN EXCITED DIATOMIC GAS

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The influence of vibrational relaxation on the nonlinear evolution of a large vortex structure in a shear flow of a highly nonequilibrium diatomic gas is studied. Calculations are performed using the equations of two-temperature gas dynamics for a viscous heat-conducting gas. Relaxation of the temperature of vibrational levels of gas molecules to equilibrium is described by the Landau–Teller equation. The contribution of the relaxation of rotational levels is taken into account by the bulk viscosity in the stress tensor. It is shown that in the presence of only the relaxation process with no viscous dissipation, the damping of the kinetic energy of perturbations and Reynolds stresses increases by up to 10% *compared to the case of thermal equilibrium. For high* (*actually attainable*) *degrees of excitation of the vibrational mode, moderate dynamic and bulk viscosities, and a typical relaxation time comparable to flow time, the relative effect of perturbation damping reaches* 15%.

Key words: *vortex structure, vibrational mode, excitation, relaxation, perturbation damping.*

Introduction. The influence of moderate thermal nonequilibrium of internal degrees of freedom on the nonlinear evolution of a large vortex structure in a shear flow of a molecular gas was studied in [1]. In this case, the energy distribution among dynamic modes was characterized by a unified temperature and the model flow was calculated using the complete Navier–Stokes equations for a compressed heat-conducting gas, in which thermal nonequilibrium was taken into account by the bulk viscosity in the divergent part of the stress tensor. According to the calculation results, as the bulk viscosity increases in the range of its real values, the mean rate of damping of the perturbation energy and the Reynolds stress modulus increases by approximately 10% compared to the case where the bulk viscosity is eliminated, for example, by using the Stokes relation [2]. The chosen structure and model flow parameters allow this result to be regarded as pulsation damping in a laminar–turbulent transition or in a developed turbulent flow. This can lead to an increase in the transition Reynolds number in the first case and to a decrease in turbulent resistance in the second case. In this connection, it is of interest to further examined the effect obtained. In the present paper, the case of a greater departure from equilibrium with excitation of the lower vibrational levels of molecules is examined invoking the vortex perturbation model used in [1]. Such nonequilibrium can be produced in a natural way, for example, during gas expansion in a nozzle, or by an artificial excitation, in particular, by moderate laser pumping.

1. Physical Model. The choice of a method to describe nonequilibrium over internal degrees of freedom of molecules depends on the relations among the mean free time of molecules τ_{tt}, the relaxation times of rotational modes τ_{rt} and vibrational modes τ_{vt}, and the characteristic flow time t_f. If the gas flow is generally considered in a hydrodynamic approximation, two alternative models of vibrational relaxation corresponding to the microscopic and macroscopic approaches are possible [2, 3]. The microscopic approach uses the equations of level kinetics describing the time evolution of the populations (occupation numbers) of vibrational levels of molecules. Since in the present paper, we study the fundamental possibility of the damping effect of relaxation, it is expedient to use a reduced

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description of the process at the macroscopic level. For this, we confine ourselves to the best-studied diatomic gases, whose molecules have only one vibrational mode.

Let us consider the widely occurring case of aerodynamics where the characteristic times satisfy the conditions $\tau_{tt} \leqslant \tau_{rt} \ll \tau_{vv} \ll \tau_{vt} \sim t_f$ [4–6]. Here τ_{vt} is the characteristic time of exchange of vibrational quanta between molecules during which a quasiequilibrium distribution with a vibrational temperature different from the flow temperature is established in the subsystem of vibrational levels. It can be assumed in this case that at times of the order of the flow time t_f, the translational and rotational degrees of freedom are in equilibrium and are characterized by a static flow temperature.

An adequate physical model for such flows is two-temperature gas dynamics, in which the contribution of the rotational motion of molecules is taken into account by the bulk viscosity μ_b in the stress tensor and the energy exchange between the vibrational mode and quasiequilibrium degrees of freedom is described by the relaxation Landau–Teller equation [3, 4]. In the spatially homogeneous case, this equation has the characteristic form

$$\frac{dT_{\text{vib}}}{dt} = \frac{T - T_{\text{vib}}}{\tau_{vt}},$$

where T is the equilibrium gas temperature and T_{vib} is the vibrational temperature. The equation describes the asymptotic process of approach of the vibrational mode to equilibrium in an average sense.

Similar relaxation equations for vibrational modes were derived by many authors using various kinetic approaches: the equations of level kinetics for a harmonic oscillator model [2, 3], the Ulenbeck–Wang-Chang kinetic equation using the generalized Chapman–Enskog method [5], the Ulenbeck–Wang-Chang kinetic equation using the Grad moment method [6]. Each of these theoretical approaches have limitations but in practice, the range of applicability of the equations is extended by using experimental data on relaxation times. This makes it possible to employ the model of two-temperature (multitemperature) gas dynamics beyond theoretical limitations in numerous calculations of hypersonic flows and discharges, shock-wave processes, and the internal aerodynamics of jet engines and lasers (see [2, 3, 5, 7] and the references therein).

A real physical factor that can force one to pass from the Landau–Teller equation to the microscopic model of level kinetics is the development of anharmonicity of the vibrational mode for a high degree of its excitation. In this connection, it is necessary to specify the ranges of physical parameters for calculations using the two-temperature model.

1.1. *Degree of Excitation and Relaxation Times of Vibrational Modes.* As a crude estimate [7], it is generally agreed that the Landau–Teller equations give satisfactory results up to the temperature $T \sim \Theta = h\nu/k$, where Θ is the characteristic temperature (the energy of the main vibrational quantum in the absolute scale) above which anharmonicity effects begin to manifest themselves. For diatomic gases, such as nitrogen, oxygen, and carbon monoxide, it is high enough and takes values $\Theta_{\text{N}_2} = 3400$ K, $\Theta_{\text{O}_2} = 2300$ K, and $\Theta_{\text{CO}} = 3100$ K [7], respectively. Although the absolute values of gas temperatures do not appear in the subsequent calculations, for definiteness, we assume $T \leqslant 2000$ K for all versions of initial data. We note that at this temperature, the degree of dissociation in these gases does not exceed fractions of percent.

It is generally agreed that for simple diatomic molecules, anharmonicity always implies excitation of a large number of vibrational levels. The degree of excitation of the vibrational mode is estimated by the ratio

$$\xi = (T_{\text{vib}} - T)/T.$$

In our case, we should confine ourselves to the level of excitation for which the anharmonicity of vibrations leading to a marked molecular dissociation can be ignored. Beforehand, it is necessary to estimate the possible range of initial values of the parameter ξ attainable by a particular method.

For a rapid nozzle gas expansion or an underexpanded jet discharge, it can be approximately assumed that the vibrational temperature is frozen at the retardation temperature. In this case, we obtain the estimate

$$\xi = (\gamma - 1)\text{M}^2/2,$$

where M is the flow Mach number and γ is the adiabatic exponent. From this, for diatomic gases it follows that for Mach numbers M = 0.1–5, the degree of excitation of the vibrational mode of molecules is in the range $\xi \approx 0$–5. This estimate is supported by detailed calculations [4] using multitemperature gas dynamics, where for air at M = 4.5 and static temperature $T = 216$ K, the value $\xi \approx 3.82$ is obtained.

The possibility of optical pumping of the vibrational mode using a laser of an appropriate wavelength is of interest. In this case, the degree of excitation ξ is generally determined by the radiation power and wavelength and by the optical properties of the gas. Experimental data (see [3, 8–10]) show that at a normal temperature and pressure and heat flux radiation wavelengths of 20 to 200 nm, the degree of excitation of the vibrational mode of molecules is in the range $\xi \approx 4.8$–10.2 for N_2, $\xi \approx 4.3$–10 for O_2, and $\xi \approx 6.2$–9.4 for CO. Thus, in all cases, the degree of molecular dissociation does not exceed 1%. In papers devoted to anharmonicity effects (see the references in [3, 7] and, e.g., [11]), the nonequilibrium criterion for the vibrational mode is the ratio $\xi_1 = T_1/T$, where $T_1 = \Theta/\ln(x_0/x_1)$ is a conditional quantity called the "temperature" of the first vibrational level and x_0 and x_1 are the populations of the zero and first levels. According to the calculations of [11], substantial development of anharmonicity, in particular, in heat capacities and transfer coefficients, begins with $\xi_1 > 4$ and is observed to $\xi_1 \approx 10$ at gas temperatures $T \approx 1000$ K, after which it is necessary to allow for dissociation. At the same time, the ratio ξ includes the $T_{\rm vib}$, which has the meaning of the vibrational energy averaged over all levels. This implies that $T_{\rm vib} > T_1$ or even $T_{\rm vib} \gg T_1$, depending on excitation. Thus, $\xi > \xi_1$, and for optical pumping of the vibrational energy levels of diatomic gases, it is possible to ignore anharmonicity if the degree of excitation is limited by the range $\xi \approx 4$–5.

The possible times of vibrational relaxation τ_{vt} were estimated by the semiempirical dependence

$$\ln(p\tau_{vt}) = Cm_{\rm eq}^{1/2}\Theta^{4/3}(T^{-1/3} - 0.015m_{\rm eq}^{1/4}) - 18.420,$$

which generalizes numerous experimental data for diatomic gases in the temperature range $T = 3 \cdot 10^2$–10^4 K. Here τ_{vt} is in seconds, the gas pressure p in atmospheres, the gas temperature T and the characteristic vibrational temperature Θ in Kelvin, and the normalized mass of molecules during collisions $m_{\rm eq}$ in atomic units. The values of the characteristic temperature Θ were given above. According to the data of [12], the constant C has the following values: $C_{\rm N_2} = 1.15 \cdot 10^{-3}$, $C_{\rm O_2} = 1.10 \cdot 10^{-3}$, and $C_{\rm CO} = 1.05 \cdot 10^{-3}$. Calculations for these gases at a pressure $p = 1$ atm and gas temperatures T of 300 to 1000 K show that the vibrational-relaxation times are in the range $\tau_{vt} \approx 1$–10^{-4} sec and decrease with increases in gas temperature.

It should be noted that the above dependence approximates experimental data by straight lines in the so-called Landau–Teller coordinates $[\ln(p\tau_{vt}), T^{-1/3}]$ [3], which indirectly confirms the possibility of using them in relaxation equations.

Losev [3] introduced a phenomenological correction to the relaxation time that permits extending the range of applicability of the Landau–Teller equation to take into account the anharmonicity of vibrations. The relaxation time is given by

$$\tau_a = \tau_{vt}\Big[\frac{1 - \gamma\exp(-\Theta/T_{\rm vib})}{1 - \exp(-\Theta/T_{\rm vib})}\Big]^2.$$

It is obvious that the thus corrected relaxation time depends not only on the gas temperature but also on the vibrational temperature $T_{\rm vib}$. It is of interest to estimate to the vibrational temperatures of the modeled gases up to which this correction can be used and its value in the range of calculated parameters.

An estimate the range of applicability for $T_{\rm vib}$ follows [3] from the obvious inequality $1-\gamma\exp(-\Theta/T_{\rm vib}) > 0$, whence $T_{\rm vib} < \Theta/\ln\gamma$. For N_2, O_2, and CO molecules, we have the approximate expression $\ln\gamma \simeq 1.87\Theta^{2/3}T^{-1/3}x_e$, where $x_e = \Theta/(4D)$ is the anharmonicity parameter and D is the dissociation energy in Kelvin. Setting the mean values of $\Theta \simeq 2500$ K and $D \simeq 10^5$ K for the modeled gases at $T \simeq 2000$ K, we find that the correction is applicable in the range $T_{\rm vib} < 1.5 \cdot 10^4$ K. In this case, the minimum degree of excitation is estimated as $\xi < (1.5 \cdot 10^4 - 2000)/2000 = 6.5$. For the same mean parameter values and $\Theta \simeq T_{\rm vib}$, the anharmonicity correction is approximately equal to

$$\Big[\frac{1 - \gamma\exp(-\Theta/T_{\rm vib})}{1 - \exp(-\Theta/T_{\rm vib})}\Big]^2 \simeq \Big[\frac{1 - 1.18\ e^{-1}}{1 -\ e^{-1}}\Big]^2 \simeq 0.82.$$

It can be assumed that this correction is in the range of spread of experimental data [12].

1.2. *Transfer Coefficients and Flow Parameters.* In the momentum and energy equations of two-temperature gas dynamics, the stress tensor P and the heat flux vector $\boldsymbol{q}$ have the form [4, 6]

$$P = pI - \mu(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^* - (2/3)I\,\mathrm{div}\,\boldsymbol{u}) - \mu_b I\,\mathrm{div}\,\boldsymbol{u},$$

$$\boldsymbol{q} = -\lambda\nabla T - \lambda_{\rm vib}\nabla T_{\rm vib}, \qquad \lambda = \lambda_{\rm tr} + \lambda_{\rm rot}, \qquad p = \rho RT,$$

where $\boldsymbol{u} = (u, v)$ is the velocity; I is a unit second-rank tensor; the asterisk denotes transposition; μ and μ_b are dynamic and bulk viscosities; $\lambda_{\rm tr}$, $\lambda_{\rm rot}$, and $\lambda_{\rm vib}$ are the thermal conductivities due to translational, rotational, and vibrational motions, and R is the gas constant. The bulk viscosity μ_b in the stress tensor P and the thermal conductivities $\lambda_{\rm rot}$ in the heat flux vector $\boldsymbol{q}$ take into account the contributions of rotational degrees of freedom, and the thermal conductivity $\lambda_{\rm vib}$ takes into account the contribution of the vibrational mode.

As in [1], the ratio of bulk viscosity to shear viscosity was estimated in the range $\alpha_1 = \mu_b/\mu \approx 0$–$2$. The thermal conductivities due to the different types of degree of freedom were determined from the following formulas taking into account the modified Eucken corrections (see [4]):

$$\lambda_{\rm tr} = 5\gamma_{\rm tr}\mu c_v/2, \qquad \lambda_{\rm rot} = 6\gamma_{\rm rot}\mu c_v/5, \qquad \lambda_{\rm vib} = 6\gamma_{\rm vib}\mu c_v/5.$$

Here $\gamma_{\rm tr}$, $\gamma_{\rm rot}$, and $\gamma_{\rm vib}$ are the fractions of internal energy contained in the translational, rotational, and vibrational degree of freedom, respectively and c_v is the total specific heat in constant volume. For a diatomic gas at $T \leqslant 2000$ K, for simplicity, we set $\gamma_{\rm rot} = \gamma_{\rm vib} \approx 2/7$ and $\gamma_{\rm tr} \approx 3/7$. The thermal conductivity ratio used in the calculations was constant $\alpha_2 = \lambda_{\rm vib}/\lambda \approx 0.2424$, where $\lambda = \lambda_{\rm tr} + \lambda_{\rm rot}$. The adopted approximation is also justified by the fact that the main objective of the calculations was to elucidate the damping effect against the background of a purely relaxation process, in which all dissipative coefficients vanished: $\mu = \mu_b = \lambda_{\rm tr} = \lambda_{\rm rot} = \lambda_{\rm vib} = 0$.

The model flow was considered in a square with side l on the plane (x, y). The carrier flow had an antisymmetric linear velocity profile $u(y) = 2U_0 y/l$. The initial vortex perturbation was specified as a Rankine vortex of radius R_0 with constant vorticity Ω_0 centered in the calculated region. On the external boundaries of the region parallel to the carrier flow, constants temperature T_0 and density ρ_0 were specified.

2. Formulation of the Problem and Method of Solution. 2.1. *Initial-Boundary-Value Problem.* The structure evolution in the model mesh was described by the complete system of equations of two-temperature gas dynamics for a viscous heat-conducting gas. The equations were normalized by the following characteristic quantities: the initial diameter of the structure $2R_0$ the velocity modulus U_0, the density ρ_0 and temperature T_0 on the upper and lower boundaries of the model mesh, time $t_0 = 2R_0/U_0$, and the pressure $p_0 = \rho_0 U_0^2$.

In the dimensionless variables, the system of equations is written as

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho\boldsymbol{u} = 0,$$

$$\rho\Big(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u}\Big) = -\nabla p + \frac{1}{\rm Re}\,\Delta\boldsymbol{u} + \frac{1}{\rm Re}\Big(\alpha_1 + \frac{1}{3}\Big)\nabla \operatorname{div}\boldsymbol{u},$$

$$\rho\Big(\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T\Big) = (\gamma - 1){\rm M}_0^2\Big(\frac{\partial p}{\partial t} + \boldsymbol{u}\cdot\nabla p\Big) + \frac{1}{\rm Re\,Pr}\,\Delta T \tag{1}$$

$$+ \frac{(\gamma-1){\rm M}_0^2}{2\,\rm Re}\,(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^*)^2 + (\gamma-1)\Big(\alpha_1 - \frac{2}{3}\Big)\frac{{\rm M}_0^2}{\rm Re}\,(\operatorname{div}\boldsymbol{u})^2 + \frac{\gamma_v\rho(T_{\rm vib} - T)}{\tau_{vt}},$$

$$\gamma_v\rho\Big(\frac{\partial T_{\rm vib}}{\partial t} + \boldsymbol{u}\cdot\nabla T_{\rm vib}\Big) = \frac{\alpha_2}{\rm Re\,Pr}\,\Delta T_{\rm vib} + \frac{\gamma_v\rho(T - T_{\rm vib})}{\tau_{vt}}, \qquad \gamma{\rm M}_0^2 p = \rho T.$$

In system (1), the equation for the vibrational temperature $T_{\rm vib}$ is a relaxation Landau–Teller equation written in Euler form [4, 6]. The first term on the right side of this equation describes the local flux of vibrational energy due to molecular transfer, and the second term describes the vibrational energy relaxation to equilibrium.

On the mesh boundaries at all times, the following conditions were specified:
for $x = \pm\chi/2$ and $y \in [-\chi/2; \chi/2]$,

$$u(\chi/2, y, t) = u(-\chi/2, y, t), \qquad v(\chi/2, y, t) = -v(-\chi/2, y, t),$$

$$\rho(\chi/2, y, t) = \rho(-\chi/2, y, t), \qquad p(\chi/2, y, t) = p(-\chi/2, y, t), \tag{2}$$

$$T_{\rm vib}(\chi/2, y, t) = T_{\rm vib}(-\chi/2, y, t);$$

for $y = \pm\chi/2$ and $x \in [-\chi/2; \chi/2]$,

$$u(x, \chi/2, t) = -u(x, -\chi/2, t), \qquad v(x, \chi/2, t) = v(x, -\chi/2, t),$$

$$\rho(x, \chi/2, t) = \rho(x, -\chi/2, t), \qquad p(x, \chi/2, t) = p(x, -\chi/2, t), \tag{3}$$

$$T_{\rm vib}(x, \chi/2, t) = T_{\rm vib}(x, -\chi/2, t).$$

In the calculation domain, the carrier flow region is specified as the exact steady-state solution of system (1) for the case of thermal equilibrium over the degrees of freedom with boundary conditions (2) and (3). In the dimensionless variables, the flow characteristics have the form

$$\begin{gathered} T_0(y) = T_{\text{vib},\,0}(y) = 1 + (\gamma - 1)\mathrm{M}_0^2\mathrm{Pr}\ (1 - 4y^2/\chi^2)/2, \\ U(y) = 2y/\chi, \qquad V = 0, \qquad \rho_0(y) = T_0^{-1}(y), \qquad P_0(x,\,y) = 1/(\gamma\mathrm{M}_0^2). \end{gathered} \tag{4}$$

The initial conditions for the velocity field and thermodynamic quantities were specified as

$$\begin{gathered} u(0,x,y) = \begin{cases} U(y) + \beta y/(2r^2), & r > 1/2, \\ U(y) + 2\beta y, & r \leqslant 1/2, \end{cases} \\ v(0,x,y) = \begin{cases} -\beta x/(2r^2), & r > 1/2, \\ -2\beta x, & r \leqslant 1/2, \end{cases} \\ T(0,x,y) = T_0(y), \quad T_{vib}(0,x,y) = (1+\xi)T_0(y), \quad \rho(0,x,y) = \rho_0(y), \quad r = \sqrt{x^2+y^2}. \end{gathered} \tag{5}$$

The calculations were performed for the following parameter values: Mach number $\mathrm{M}_0 = U_0/\sqrt{\gamma R T_0} = 0.5$, Reynolds number $\mathrm{Re} = 2U_0R_0\rho_0/\mu = 100$, Prandtl number $\mathrm{Pr} = \mu c_p/\lambda = 0.74$, relative intensity of vortex perturbation $\beta = \Omega_0 R_0/(2U_0) = 0.2$, intermittency parameter $\chi = l/(2R_0) = 3$, $\gamma = 1.4$, $\alpha_1 = 0$–2, $\alpha_2 = 0.2424$, $\xi = 0$–5, $\tau_{vt} = 0$–5.

2.2. *Difference Schemes.* In the numerical calculations, system (1) was approximated by a weight finite-difference scheme with splitting in physical processes and space coordinates similar to that used in [1]. The scheme is written in operator form as

$$\frac{\boldsymbol{x}^{n+1} - \boldsymbol{x}^n}{\Delta t} + \boldsymbol{L}_h[\delta\boldsymbol{x}^{n+1} + (1-\delta)\boldsymbol{x}^n] = \boldsymbol{G}_h^n, \tag{6}$$

where $\boldsymbol{x}^n = (\rho_{ij}^n, u_{ij}^n, v_{ij}^n, T_{ij}^n, T_{\text{vib},ij}^n)$ is a grid vector function of the solution in the nth time layer, h and Δt are the spatial grid step and the step in time, respectively, and δ is the weight parameter. The form of the operator $\boldsymbol{L}_h$ is described in [1] and the right side vector $\boldsymbol{G}_h^n$ consists of second-order approximations symmetric over each coordinate with mixed derivatives from the momentum equations, the terms of the dissipative function from the energy equation, and the source terms from the relaxation equation and energy equation. In the quasiequilibrium case, where system (1) becomes the complete system of Navier–Stokes equations, scheme (6) coincides with the scheme proposed in [13] (see also [1]). An analysis shows that the appearance of an additional equation and a source term in the energy equation does not affect the computations. Thus, scheme (6) on a regular grid with a step h on both coordinates approximates system (1) with order $O(\Delta t + h^2)$ and is absolutely stable for a weight parameter $\delta > 1/2$.

To estimate the contribution of the relaxation process, the model problem in question was also solved in an approximation of relaxation gas dynamics, in which the momentum and energy equations and the relaxation equation of system (1) do not contain dissipative terms ($\mu = \mu_b = 0$ and $\lambda_{\text{tr}} = \lambda_{\text{rot}} = \lambda_{\text{vib}} = 0$). In this case, the system of equations was approximated by a weight finite-difference scheme with splitting in physical processes and space coordinates. It was constructed on the basis of a numerical scheme for the equations of gas dynamics from [13]. In abstract operator form, the scheme is also written in the form (6). However, in this scheme, the operator $\boldsymbol{L}_h$ is composed of symmetric second-order approximations of the first derivatives with respect to each space coordinate, and the right-side vector $\boldsymbol{G}_h^n$ consists of the relaxation terms from the energy equation and the relaxation equation. The approximation order and the stability of the thus obtained scheme are similar to the characteristics of scheme (6).

In the computation domain in both cases, the grid contained $31 \times 31 = 961$ nodes with a step $h = 0.1$, and the step in time was $\Delta t = 0.01$. The evolution of the perturbation was traced up to its entry to the mesh boundary, which required up to 600 time steps, after which the calculation was terminated.

To improve the numerical schemes, we performed test calculations similar to those carried out in [1]. As was shown, the maximum calculation errors do not exceed $5 \cdot 10^{-3}$.

3. Calculation Results and Discussion. In the calculations, we examined the energy and momentum transfer between the imposed perturbation and the carrier flow. The pulsation flow characteristics Φ' were defined by

$$\Phi' = \Phi - \Phi_0,$$

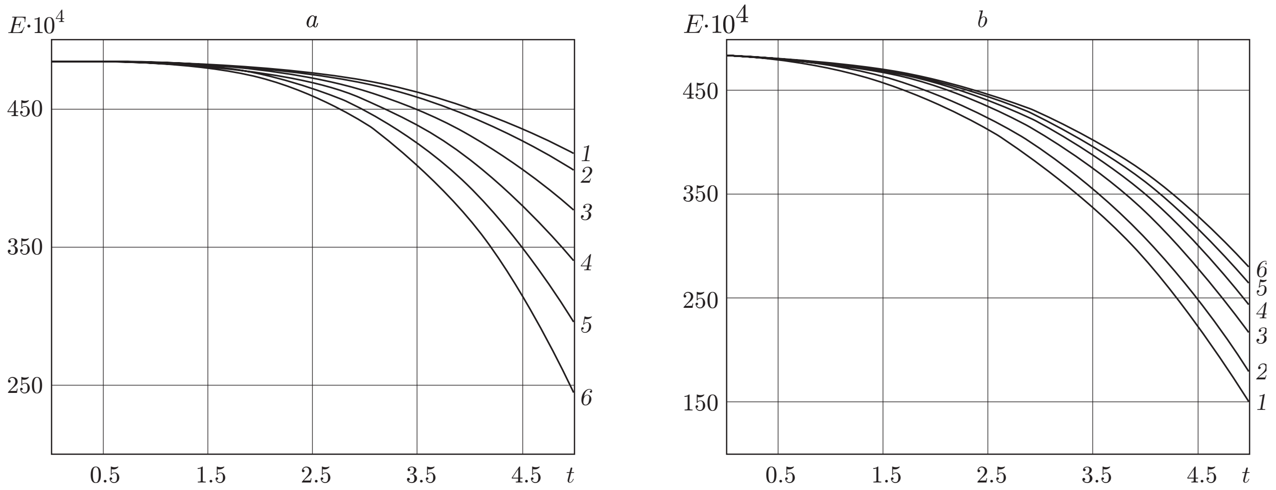

Fig. 1. Kinetic energy of perturbation versus time (Re $= 100$, $\mathrm{M}_0 = 0.5$, Pr $= 0.74$, $\beta = 0.2$, $\chi = 3$, $\alpha_1 = 0$–2, $\xi = 0$–5, and $\tau_{vt} = 0$–5): (a) $\mu = \mu_b = 0$, $\lambda = \lambda_{\text{vib}} = 0$, $\tau_{vt} = 3$, and $\xi = 0.5$ (1), 1 (2), 2 (3), 3 (4), 4 (5), and 5 (6); (b) $\alpha_1 = 0.5$, $\xi = 2$, $\tau_{vt} = 0.5$–5, and $\tau_{vt} = 0.5$ (1), 1 (2), 2 (3), 3 (4), 4 (5), and 5 (6).

where Φ are instantaneous values of the flow characteristics obtained by numerical solution of the model problem and Φ_0 are the characteristics of the equilibrium steady-state carrier flow (4). To estimate the effect of relaxation of excited vibrational degrees of freedom and bulk viscosity on the pulsation characteristics of the model flow, we examined the time evolution of the Reynolds stress modulus

$$\sigma_{xy}(t) = \int\limits_{-\chi/2}^{\chi/2} \int\limits_{-\chi/2}^{\chi/2} |\rho u' v'| \, dx \, dy$$

and the kinetic energy of the perturbations

$$E(t) = \frac{1}{2} \int\limits_{-\chi/2}^{\chi/2} \int\limits_{-\chi/2}^{\chi/2} \rho \boldsymbol{u}'^2 \, dx \, dy.$$

The corresponding integrals were evaluated using the rectangular formulas on a regular grid with a step $h = 0.1$.

In Fig. 1a, the plots of $E(t)$ demonstrate the effect of the purely relaxation process versus the degree of excitation of the vibrational mode ξ for the case where in Eqs. (1), the dissipative coefficients are equal to zero ($\mu = \mu_b = 0$ and $\lambda_{\text{tr}} = \lambda_{\text{rot}} = \lambda_{\text{vib}} = 0$) and the vibrational relaxation time $\tau_{vt} = 3$ is comparable to the characteristic time of flow evolution. The course of the curves shows that as the value of ξ increases, the dependences $E(t)$ decrease more rapidly. The effect of the variation in the vibrational relaxation time τ_{vt} on the damping of the perturbation energy $E(t)$ is illustrated in Fig. 1b. The parameter τ_{vt} varies within one order of magnitude. The values of the parameters α_1 and ξ, which also influence dissipation, are moderate. It is obvious that with decrease in the relaxation time, the pulsations are damped more rapidly, although not in a direct proportion to the variation of τ_{vt}. The dependences $\sigma_{xy}(t)$ for various ξ and τ_{vt} behave same as the curves presented in the figure. As the degree of excitation of the vibrational energy levels ξ increases, the Reynolds stresses damp more rapidly, and a decrease in the vibrational relaxation time τ_{vt} leads to an increase in the damping rate $\sigma_{xy}(t)$.

The effect of the vibrational relaxation process on the mean pulsation characteristics was estimated quantitatively by calculating their relative changes:

$$\Delta_F^{\xi} = |\langle F(\xi, \tau_{vt}) \rangle - \langle F(0, \tau_{vt}) \rangle| / \langle F(0, \tau_{vt}) \rangle, \qquad \tau_{vt} = \text{const},$$

$$\Delta_F^{\tau_{vt}} = |\langle F(\xi, \tau_{vt}) \rangle - \langle F(\xi, 0) \rangle| / \langle F(\xi, 0) \rangle, \qquad \xi = \text{const}.$$

The averaging in time was defined by

$$\langle F \rangle = \frac{1}{\theta} \int\limits_0^{\theta} F(t) \, dt.$$

For the regime shown in Fig. 1, in an approximation of relaxation gas dynamics with $\mu = \mu_b = 0$, $\lambda_{\rm tr} = \lambda_{\rm rot} = \lambda_{\rm vib} = 0$, $\tau_{vt} = 3$, these characteristics for the kinetic energy $\langle E\rangle$ and the Reynolds stress modulus $\langle\sigma_{xy}\rangle$ for $\xi = 0$–5 vary in the range $\Delta_F^{\xi} = 0.043$–0.091, and for $\tau_{vt} = 5$, they vary in the range $\Delta_F^{\xi} = 0.017$–0.056. For vibrational relaxation times $\tau_{vt} = 0$–5 and fixed values $\alpha_1 = 0.5$ and $\xi = 2$, the relative variations of the values of $\langle E(\xi, \tau_{vt})\rangle$ and $\langle\sigma_{xy}(\xi, \tau_{vt})\rangle$ are in the range from $\Delta_F^{\tau_{vt}} = 0.103$ for $\tau_{vt} = 0.5$ to $\Delta_F^{\tau_{vt}} = 0.01$ for $\tau_{vt} = 5$. In the case where the bulk viscosity reached the maximum value adopted in calculations (see also [1]) and corresponding to $\alpha_1 = 2$, for $\xi = 2$, we obtained $\Delta_E^{\tau_{vt}} = 0.158$ for $\tau_{vt} = 3$ and $\Delta_E^{\tau_{vt}} = 0.106$ for $\tau_{vt} = 5$.

Conclusions. The effect of the relaxation of vibrational levels of molecules on the nonlinear interaction of a vortex perturbation of finite amplitude with a carrier shear flow of an excited diatomic gas was investigated numerically. The ranges of the degree of excitation and relaxation time of the vibrational mode and bulk viscosity corresponded to their real values for nitrogen, oxygen, and carbon monoxide. The results of the numerical modeling suggest that the nonequilibrium of the vibrational mode of the gas molecules has a marked damping effect on the perturbation dynamics at the excitation levels attainable in nozzle flows, underexpanded jets or moderate laser pumping.

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